Basic Definition

A Geometry is a mathematical system consisting of a set of points P, collections of points called lines, an angle measurement function, and a distance function.

Euclidean Geometry

Line Segment

 Draw line segments with a length of 5 – vertical, horizontal, neither

 Find the midpoint of each segment

 Now draw in the perpendicular bisector of each segment

Distance to City Hall

 City Hall (-2,2)

 Post Office (1,2)

 Museum (3,4)

 Which is closer to the Post Office?

Circle with a radius of 3

Circles: center (-4,2) r=3 and (2,-2) r=7 sketch the intersection.

Drinking fountain problem

A water fountain company wants to put water fountains in a 12 by 12 block park so that wherever people are, they are within two blocks of a water fountain. Money is tight and the company wants to know where the water fountains should be placed so they can be the most cost effective.

Ali and Bryce problems Ali works @ (-3,-1) Bryce works @ (3,3)

 Where can they live so that the sum of their distances is a minimum

 Where can they live so that their distances would be the same

 Where can they live so that Bryce walks farther

 Bryce starts work at 5:30 Ali at 8 – so Bryce wants to live closer

Cement Company

 A: sand quarry at (-6,0)

 B: boat dock at (5,0)

 C: railroad freight yard at (-2,5)

 Locate P:plant so that the sum of the distances to A, B, and C is a minimum

Elementary School Problem

 Three elementary schools are located at

 A: (-3,-1) B:(3,3) C:(0,-4)

Partition the grid to determine where students should go to school based upon their home locations

What is PI?

 Indiana didn’t like it so tried to legislate it to be 3.14

What do conic sections look like?

 Circle, ellipse, parabola, hyperbola

What does a sin curve look like (radian measure)?

Taxi-Cab Geometry

Taxi-cab geometry is metric where you can only make right angle turns to move.

What is taxi-cab geometry? How is it different from Euclidean?

 dE(a,b) = $\sqrt{(x\_{a}-x\_{b})^{2}+(y\_{a}-y\_{b})^{2}}$

 dT(ab) = $|(x\_{a}-x\_{b)}|+$ |$\left(y\_{a}-y\_{b}\right)|$

How does this affect our previous answers?

Line Segment

 What is the definition?

Given collinear points A, B, and C, C is between A and B if AC + BC = AB.

The line segment $\overbar{AB} $consists of endpoints A and B and all points between A and B.

… now

 Draw line segments with a length of 5 – vertical, horizontal, neither

 Define a midpoint:

 Find the midpoint of each segment

 Define a perpendicular bisector

 Now draw in the perpendicular bisector of each segment

Draw the perpendicular bisector to the segment from A(2,2) B(4,4)

Distance to City Hall

 City Hall (-2,2)

 Post Office (1,2)

 Museum (3,4)

 Which is closer to City Museum?

Circles

 What is the definition?

A circle is the set of all points in a plane that are equidistant from a given point in the plane.

Draw a circle with a radius of 3

Circles: center (-4,2) r=3 and (2,-2) r=7 sketch the intersection.

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What is PI?

 Define PI:

 Indiana didn’t like it so tried to legislate it to be 3.14

What do conic sections look like?

 Circle

Define Circle:

Ellipse

Define Ellipse:

Parabola

Define Parabola:

Hyperbola

Define hyperbola:

What does a sin curve look like (radian measure)?

Conic Sections

A circle is the set of all points in a plane that are equidistant from a given point in the plane.

An ellipse is the set of all points in a plane the sum of whose distances from two given points (foci) is constant.

A hyperbola is the set of all points in a plane the difference of whose distances from two given points (foci) is constant.

A parabola is the set of all points in a plane equidistant from a given point (focus) and a given line (directrix).

Other definitions

Π is defined as the ratio of the circumference of a circle to its diameter.

A geometry is called Euclidean if it satisfies the following thirteen properties:

1. Given any two points there is exactly one line containing them.
2. Every line contains at least two points, P contains at least 3 non-collinear points.
3. The distance between two points is a non-negative number and is zero only if the two points coincide.
4. The distance from A to B equals the distance from B to A.
5. The distance function satisfies the triangle inequality property.
6. Given any line, there exists a one-to-one and onto function that acts as a ruler for all pairs of points A, B on the line.
7. Each line divides the space into two convex half-planes.
8. The angle measurement function assigns to each angle a real number between 0 and 180.
9. Given a ray $\vec{AB}$ on the edge of a half-plane and given a real number r between 0 and 180, there is exactly one ray extending from A into the half-plane with the angle measure of r.
10. The angle addition property.
11. Two angles forming a straight angle sum to 180.
12. The side-angle-side congruence holds for triangles.
13. The parallel postulate: Given a point A not on a line *l*, there is exactly one line through A parallel to *l*.

Taxicab geometry

Euclidean metric: dE(A,B) =$\sqrt{(x\_{A}-x\_{B})^{2}+(y\_{A}-y\_{B})^{2}}$

Taxicab metric; dT(A,B) = | xA – xB | + | yA – yB |

1. Plot P and Q, find dE(P,Q) and dT(P,Q)

|  |  |  |  |
| --- | --- | --- | --- |
| P | Q | dE(P,Q) | dT(P,Q) |
| (5,4) | (1,2) |  |  |
| (-4,3) | (3,2) |  |  |
| (-5,-4) | (1,-2) |  |  |
| (3,-1) | (-2,4) |  |  |
| (4,-3) | (-2,3) |  |  |

1. If dT(A,B) = dT(C,D) then dE(A,B) = dE(C,D). True / False

If dE(A,B) = dE(C,D) then dT(A,B) = dT(C,D). True / False

Under what conditions on A and B does dT(A,B) = dE(A,B)?

dE(A,B) ≤ dT(A,B) ? True / False

1. A = (-2,-1) Plot A.

Plot each point P and then find dT(P,A).

|  |  |  |  |
| --- | --- | --- | --- |
| P | dT(P,A) | P | dT(P,A) |
| (1,-1) |  | (.5,-1.5) |  |
| (-2,-4) |  | (-1.5,-3.5) |  |
| (-1,-3) |  | (0,0) |  |
| (0,-2) |  | (-2,2) |  |

1. A = (-2,-1) Find more points P such that dT(P,A) = 3.

Graph the set of all points P such that dT(P,A) = 3

 i.e. { P | dT(P,A) = 3 }

Graph { P | dE(P,A) = 3 }

What is a reasonable name for { P | dT(P,A) = 3 }?

In taxicab geometry π = ?

1. A = (-2,-1) B = (3,2)

Graph A and B

Graph a taxicab circle, center A radius 2

Graph { P | dT(P,A) = 1 }

Graph the set of all points at a taxicab distance 1.5 from A

Graph { P | dT(P,A) = 2.5 }

1. A = (-2,-1) B = (3,2)

Calculate dT(A,B) =

Graph { P | dT(P,A) = 3 } and { P | dT(P,**B**) = 5 }

Graph { P | dT(P,A) = 1 } and { P | dT(P,**B**) = 7 }

Graph { P | dT(P,A) = 0 } and { P | dT(P,B) = 8 }

Graph { P | dT(P,A) = 1.5 } and { P | dT(P,B) = 6.5 }

Graph { P | dT(P,A) = 4 } and { P | dT(P,B) = 4 }

Graph { P | dT(P,A) = 5 } and { P | dT(P,B) = 3 }

Graph { P | dT(P,A) + dT(P,B) = dT(A,B) }

1. A = (-7,-3) and B = (5,2)

Calculate dE(A,B)

Graph { P | dE(P,A) + dE(P,B) = dE(A,B) }

1. On a sheet of graph paper mark each pair of points A and B and then graph

{ P | dT(P,A) + dT(P,B) = dT(A,B) }

A = (-2,3) B = (1,-4) A = (1,-3) B = (4,0)

A = (2,1) B = (6,1) A = (1,1) B = (1,4)

1. A = (-2,-1) B = (3,2)

Graph { P | dT(P,A) = 5 and dT(P,B) = 5 }

Graph { P | dT(P,A) = 7 and dT(P,B) = 7 }

Graph { P | dT(P,A) = 4 and dT(P,B) = 4 }

Graph { P | dT(P,A) = dT(P,B) }

1. Repeat 9 using dE in place of dT.
2. For each pair of points A and B, graph { P | dT(P,A) = dT(P,B) }

A = (0,0) B = (4,2) A = (0,0) B = (2,4)

A = (0,0) B = (3,3) A = (-1,1) B = (4,1)

1. Plot A = (-3,0) and B = (1,2) and then graph { P | dT(P,A) = 2 · dT(P,B) }
2. Repeat 12 using dE in place of dT.

Taxicab Geometry Applications

Taxicab geometry is a mathematical model for urban geography. It is not a perfect model, but it is a better model than Euclidean geometry – why? Some of the assumptions in this model are that all streets run north and south, they have zero width, all locations are points. These assumptions do not make this a bad model, you just need to recognize the assumptions. No mathematical model is a perfect representation of reality.

1. Alice and Bill are looking for an apartment in Yellowcab city. Alice works as an interior decorator at Insides R Us, location (-3,-1). Bill works as a cookie taster at the Cookie Monster bakery, location (3,3). Being ecologically aware and cheap, they walk everywhere they go. They have decided that their apartment should be located so that they distance Alice walks to work plus the distance Bill has to walk to work is as small as possible. Where should they look for an apartment?
2. In a weak moment Bill decides that the sum of the distances should still be a minimum, but Alice should not have to walk further than he does. Now where should they look?
3. Alice likes this idea, but feels it is more important that they each walk the same distance. Now where could they live?
4. After an unsuccessful day of looking for apartments, Alice and Bill decide they need to widen the search. The key requirement is that they both have to be the same distance from their jobs. Now where do they look?
5. After another luckless day they have a change of thinking and decide that Bill should be closer to his job than Alice. Now where can they look?
6. The dispatcher for the Yellowcab police department receives a report of a disturbance at X = (-1,4). There are two police cars in the area, car C at (2,1) and car D at (-1,-1). Which car should be sent to the disturbance?
7. A developer wants to put up an apartment building within six blocks of the shopping center S = (-3,0) and within four blocks of the tennis courts T = (2,2). Where should he look for land?
8. The newly elected mayor, Jim Ignitowski, has promised to install drinking fountains in Yellowcab city in a park so that every citizen is within three blocks of a drink. The park is a 12 block by 12 block park. He discovers that money for civic improvements is very scarce. Design a layout so that expenditures are a minimum.

1. The telephone company wants to set up pay-phone booths so that everyone living with in twelve blocks of the center of town is within four blocks of a pay phone. How few booths can they get by with, and where should they be located?
2. A group of students has decided to start a Junior Achievment business of cutom finishing furniture. They will buy unfinished furniture at warehouse W = (-3,2), transport it to their shop S for finishing, and then deliver it to retail store R = (5,-1) for sale. Where should they locate their shop S if they want to minimize the distance they will have to haul furniture?
3. Thre are three high schools in Yellowcab City: Fillmore at (-4,3), Grant at (2,1), and Harding at (-1,-6). Draw in school-district boundary lines so that each student in Yellowcab City attends the high school nearest his home.
4. If Burger Baron wants to open a hamburger stand equally distant from each of the three high schools, where should it be located?
5. A fourth high school, Polk High, has just been built at (2,5). Redraw the school-district boundary lines.
6. Out in UrbanSprawl, people don’t have to walk around buildings. They can walk directly (as-the-crow-flies or Euclidean distance) to their destination. Their distance function is Euclidean distance. Repeat 11-13, assuming that the schools are out on the great plains of UrbanSprawl.

Taxicab Geometry: Some Geometric Figures

The taxicab metric changes our intuitions based upon Euclidean experience. For instance a taxicab circle is different from a Euclidean circle – how? The set of all points equidistant from two given points are found on the perpendicular bisector of the segment joining the two points in Euclidean geometry. The shape is much different with the taxicab distance formula – what does the set look like?

An ellipse is the set of all points in a plane the sum of whose distances from two given points (foci) is constant.

Mark A = (-2,-1) and B = (2,2) on a sheet of graph paper.

Sketch the taxicab sets

 { P | dT(P,A) + dT(P,B) = 9 }

 { P | dT(P,A) + dT(P,B) = 13}

 { P | dT(P,A) + dT(P,B) = 7 }

 { P | dT(P,A) + dT(P,B) < 13 }

Sketch the taxicab ellipse, { P | dT(P,M) + dT(P,N) = 10 },

where M = (-2,1) and N = (4,1).

Euclidean ellipses have their applications in the heavens. Planets orbit the sun. Their path is an ellipse with the sun at one focus. Taxicab ellipses have practical uses here on earth. Alice and Bill who still work at (-3,-1) and (3,3) have decided that they want the sum of the distances that they have to walk to work should be no more than 14 blocks. Where can they look for an apartment?

Acme president, B. Bunny, wants to build a carrot canning factory in Yellowcab city. The location is to be where the sum of the distances from the railroad station C = (-5,-3) and the shipping dock D = (5, -1) is at most 16 locks. For nose-control purposes, a city ordinance forbids the location of any vegetable processing facility within 3 blocks of city hall L = (-4,2). Where can Acme build its plant?

A hyperbola is the set of all points in a plane the difference of whose distances from two given points (foci) is constant.

Mark A = (-3,-1) and B – (2,2).

Sketch { P | | dT(P,A) – dT(P,B) | = 3 }

On a new sheet of graph paper mark A = (-3,-1) and B – (2,2). Copy the taxicab hyperbola from above onto this paper. Using a different color for each figure, sketch the following

 { P | | dT(P,A) – dT(P,B) | = 1 }

 { P | | dT(P,A) – dT(P,B) | = 0 }

 { P | | dT(P,A) – dT(P,B) | = 2 }

 { P | | dT(P,A) – dT(P,B) | = 8 }

 { P | | dT(P,A) – dT(P,B) | = 9 }

What is significant about the number 8?

Investigate the family of taxicab hyperbolas with foci A = (-3,1) and B = (5,1)

Investigate the family of taxicab hyperbolas with foci A = (0,0) and B = (4,4)

Alice and Bill still don’t have an apartment. Their latest agreement is that neither person should have to walk more than 4 blocks farther to work than the other person. Where can they look?

Resources:

* Textbook – Amazon $6.95
* Taxicab Geometry by Krause
* Geometers sketchpad constructions for
	+ Segment
	+ Circle
	+ Challenges

Perpendicular bisector

Trig Curves – sin, cos, tan

